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Institute for Partial Differential Equations

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Formulary Mathematics for Engineers A
Calculus 1

Binomial theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Accumulation point p of $(a_n)_{n=0}^{\infty}$

$$\forall \varepsilon > 0 \ \forall N \ \exists n > N : |a_n - p| < \varepsilon$$

Big-$\mathcal{O}$ notation

$$a_n = \mathcal{O}(b_n) \ \Leftrightarrow \ \exists c > 0 \ \exists N : \left| \frac{a_n}{b_n} \right| \leq c \ \forall n > N$$

Common limits

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = \mathrm{e}^x$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{n!} = \infty$$

Series und series expansion

$$\mathrm{e}^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \mp \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \mp \cdots$$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \pm \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \pm \cdots$$

Injective function $f : D \to B$

$$x_1 \neq x_2, \ x_1, x_2 \in D \Rightarrow f(x_1) \neq f(x_2)$$

Surjective function $f : D \to B$

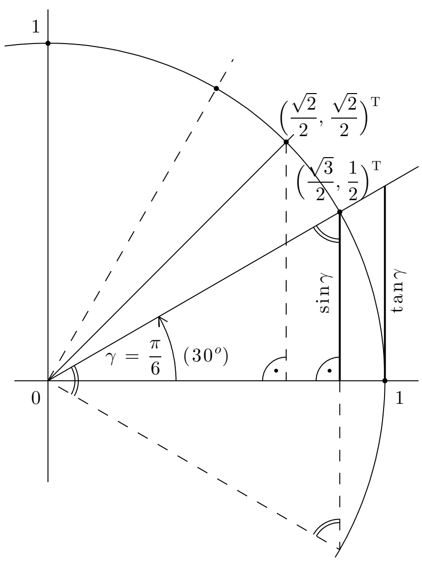
$$\forall y \in B \ \exists x \in D : f(x) = y$$

Inverse map

$$f^{-1}(y) = x \ \Leftrightarrow \ f(x) = y$$

$$\sqrt[m]{b} = a \ \Leftrightarrow \ a^m = b \ \Leftrightarrow \ \log_a b = m$$

Trigonometric functions



Euler's Identity

$$\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi$$

Trigonometric relations

$$1 = \sin^2 x + \cos^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\cos t = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t}\right)$$

$$\sin t = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}t} - \mathrm{e}^{-\mathrm{i}t}\right)$$

Hyperbolic functions

$$\cosh t = \frac{1}{2}(e^t + e^{-t})$$
$$\sinh t = \frac{1}{2}(e^t - e^{-t})$$
$$\operatorname{arsinh} t = \ln\left(t + \sqrt{t^2+1}\right)$$
$$\operatorname{arcosh} t = \ln\left(t + \sqrt{t^2-1}\right)$$

Partial fraction expansion

$$\frac{p(x)}{(x-x_0)(x-x_1)} = \frac{A}{x-x_0} + \frac{B}{x-x_1} \ ,$$

$$\frac{p(x)}{(x-x_0)^k} = \frac{A_1}{x-x_0} + \frac{A_2}{(x-x_0)^2} + \cdots + \frac{A_k}{(x-x_0)^k}$$

Continuity

$f : D \to B$ is continuous in $x_0 \in D \ \Leftrightarrow$

$\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall x, x_0 \in D :$

$|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$

Differentiation

Product rule $\quad (uv)' = u'v + uv'$

Chain rule $\quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}v} \cdot \frac{\mathrm{d}v}{\mathrm{d}x}$

$(x^n)' = n \cdot x^{n-1}, \ n \neq 0$

$(a^x)' = \ln a \cdot a^x, \ a > 0$

Derivative of the inverse map $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$

Remainder term $R_n = f(x) - T_n(x)$ to n-th Taylor polynomial T_n

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$$

with ξ inside x and x_0

Typical norms

$$||f||_{C([a,b])} = \max_{x \in [a,b]} |f(x)|$$
$$||f||_{C^1([a,b])} = \max_{x \in [a,b]} |f(x)| + \max_{x \in [a,b]} |f'(x)|$$
$$||f||_{L_2([a,b])} = \left(\int_a^b |f(x)|^2 \ \mathrm{d}x \right)^{\frac{1}{2}}$$

Integral function

$$I(x) = \int_a^x f(t) \ \mathrm{d}t$$

Integration by parts

$$\int u(x)v'(x) \ \mathrm{d}x = u(x)v(x) - \int u'(x)v(x) \ \mathrm{d}x$$

Particular integrals

$$\int_{-\infty}^{\infty} e^{-x^2} \ \mathrm{d}x = \sqrt{\pi}$$
$$\int_0^{\pi} \sin^2 x \ \mathrm{d}x = \frac{\pi}{2}$$
$$\int \ln x \ \mathrm{d}x = x \cdot \ln x - x + c \ , \ c \in \mathbb{R}$$

Selected funktions

Dirichlet-Function $\quad D(x) = \begin{cases} 1 \ , \ x \in \mathbb{Q} \\ 0 \ , \ x \notin \mathbb{Q} \end{cases}$

Heaviside-Function $\quad H(x) = \begin{cases} 1 \ , \ x > 0 \\ 0 \ , \ x \leq 0 \end{cases}$

$\Gamma -$ Function $\quad \Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} \ \mathrm{d}t$ für $x > 0$

pq formula for $x^2 + px + q = 0$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$